Bocconi University
Ph.D. in Statistics and Computer Science 2021-2022

Probability I

Probability spaces.
Definition and properties. Continuity and countably additivity. Regularity in Polish spaces.
Constructions and extensions. Kolmogorov existence theorem.
Independence of events and sigma-algebras. Limits of sequences of events. Borel Cantelli Lemmas. Kolmogorov zero-one law.
Product spaces.

Random vectors.
Random vectors and probability distributions. Measurability with respect to different sigma-algebras. Sigma-algebras generated by random vectors. Independence.
Lebesgue decomposition of probability distributions.
Transformations and convolution.

Expectation.
Definition and properties. Expectations and limits.
Inequalities.
L^p spaces.
The moment generating function

Convergence of random vectors.
Convergence in probability and almost sure convergence. Definition, properties and criteria.
Weak laws of large numbers for uncorrelated random variables or weakly correlated random variables.
Kolmogorov strong law of large numbers.

Textbook:
Billingsley *Probability and Measure.* Wiley.

Exam Rules

Written exam with open questions.