### 10.7 Dynamic analysis of partial equilibrium

The Cobweb model provides a framework for dynamic analysis of partial equilibrium. In this model it is assumed that production needs some time to realize and the amount of time needed is the same for all firms (like for example in the seasonal agricultural production). At the beginning of each production period firms choose how much to produce. The level of production is determined by the price at which firms expect to sell it in the future. The model looks at a sequence of very short term competitive equilibria of the market for the good in question. There are $t=1, \ldots$ successive periods of time in the market, $Q_{t}^{d}=D\left(p_{t}\right)$ is the (decreasing) demand curve, and $Q_{t}^{s}=S\left(p_{t}^{e}\right)$ is the (increasing) industry supply function, where $p_{t}^{e}$ stands for the firm's expectation of period $t$ price from the point of view of period $t-1$. Therefore, assumptions imposed on the expectation formation of prices influence the equilibrium. In original version of the model (see Ezekiel, 1938) it is assumed that the producers expect that price will not change over time, that is $p_{t}^{e}=p_{t-1} *$, where $p_{t-1} *$ is the equilibrium price in period $t-1$. As a result, in period $t-1$ firms choose the level of production basing their decision on current ( $t-1$ period) price, while they are selling in period $t$. The very short period market equilibrium in period $t$ can be represented by the following conditions

$$
Q_{t}^{*}=D\left(p_{t}^{*}\right)=S\left(p_{t}^{e}\right), \quad p_{t}^{e}=p_{t-1} *, \quad t=1, \ldots
$$

Let $p_{0}$ be the initial price, then these conditions determine a sequence of equilibria $\left(Q_{t}{ }^{*}, p_{t}{ }^{*}\right), t=1, \ldots$ Interesting questions to ask about this sequence are: is there convergence towards a stationary equilibrium? are there any fluctuations? From the equilibrium conditions we obtain that the price path is determined by the difference equation $D\left(p_{t}{ }^{*}\right)=S\left(p_{t-1}{ }^{*}\right)$. Let $p^{* *}$ be defined by the condition $D\left(p^{* *}\right)=S\left(p^{* *}\right)$ and let $\beta$ and $\gamma$ be the slopes defined by $\beta=-\mathrm{D}_{p} D\left(p^{* *}\right)$ and $\gamma=\mathrm{D}_{p} S\left(p^{* *}\right)$. Then, in the neighborhood of price $p^{* *}$, the preceding difference equation becomes $p_{t}^{* *}-p^{* *}=-\frac{\gamma}{\beta}\left(p_{t-1}^{*}-p^{* *}\right)$ and so we know that there is convergence towards $p^{* *}$ if $\beta>\gamma$ and divergence from it if $\beta<\gamma$. Moreover, by the assumptions $\beta>0$ and $\gamma>0$, we get that prices fluctuate.


Figure 10.17


Figure 10.18

In the figures 10.17 and 10.18 we represent these two situations graphically. (If we examined not only the price path in the proximity of the stationary equilibrium but rather global dynamics and the functions $D(p)$ and $S(p)$ were not linear, then we could also observe a convergence towards a limit cycle, that is towards persistent fluctuations between equilibria $\left(Q_{a}, p_{a}\right)$ and $\left(Q_{b}, p_{b}\right)$ where $D\left(p_{a}\right)=S\left(p_{b}\right) \quad$ and $D\left(p_{b}\right)=S\left(p_{a}\right)$ ).

As has already been indicated, the preceding analysis is based on the assumption that producers rely on the current prices to set the level of production to sell in the next period, that is on the expectation function $p_{t}^{e}=p_{t-1} *$. However, unless $p_{t-1} *=p^{* *}$, this expectation is wrong and so the conjectures that firms make are systematically incorrect. To better predict future prices firms could take into account not only present but also past prices. Then the expectation function would look like this $p_{t}^{e}=f\left(p_{t-1}{ }^{*}, p_{t-2}{ }^{*}, \ldots\right)$. We call such expectations adaptive. We could also imagine that the firms change their expectations if they perceive the presence of systematic errors. The expectations that do not suffer from this problem are called rational expectations (introduced by Muth, 1961, taking under consideration exactly the cobweb model). In the case examined here, rational expectations require that the firms know functions $D(p)$ and $S(p)$, and take $p_{t}^{e}=p^{* *}$. If the firms do not know these functions, then we need to introduce a learning process that involves observation of market prices and is subject to random shocks. That leads to results that are remarkably different from the ones stated above, from which the name cobweb model originated. (The cobweb model is also one of the first economic models that was studied empirically, as well as examined with the rational expectations hypothesis).

Note that the dynamic analysis in this paragraph differs from the analysis in Paragraph 10.3 where we examined equilibrium stability. In Paragraph 10.3 dynamics concerns the process of adjustment to equilibrium.

As this process unwinds no exchange or production takes place. Therefore we refer to the time in which the adjustment takes place as to logical time, in contrast to historical time which corresponds to economic actions. In this paragraph, on the other hand, dynamics concerns a sequence of equilibria, each of which determines the level of trade and price (effective prices, not virtual ones like the ones announced by the auctioneer). Moreover, we have a sequence of stable equilibria (in sense of Walrasian stability). The analysis aims to determine the path (in historic time) of these equilibria. In particular, we are interested whether these equilibria converge towards a stationary equilibrium, where no further modifications are observed unless a change in fundamentals occurs (in the case examined here changes in fundamentals could be changes in technology, prices of input or demand curve).

### 10.10 Monopoly equilibrium with price discrimination

In the monopoly with unique price (examined in Paragraph 10.8), the price chosen by the monopolist is the same for all buyers and for all quantities of the good bought. In the monopoly with price discrimination, the monopolist can choose different prices for different quantities of the good bought and/or for different buyers. There are different types of price discrimination. The first-degree price discrimination means that the monopolist can apply a different price for each buyer and for each unit of the good purchased. The second-degree price discrimination means that the monopolist can announce different contracts (each represented by a quantity-price pair) in the market and the buyers can choose among them. In this way the buyers pay different prices depending on the contract they chose. The third-degree price discrimination means that the monopolist distinguishes among the buyers and sells them the good at different prices (therefore, at the same price for each buyer no matter what quantity he buys).
a) Monopoly with first-degree price discrimination. In this case the monopolist can extract from each buyer the maximum price that he is willing to pay to get the good. (For example, the monopolist can be a firm producing the only available cure for a particular type of hair loss condition). This means that the buyer is forced by the monopolist to stay on his initial indifference curve no matter what quantity of the good he buys. In other words, the monopolist chooses his most preferred point on the buyer's indifference curve.

We assume that before the transaction takes place buyers do not possess the good at all. We also assume that the purchase of other goods does not depend on the conditions in the market for the good that we examine with the exception of the amount spent on the examined good. Therefore, buyer's preferences are additively separable with respect to the examined good and can be represented by a utility function of the type
$u(x)+g(m-r)$, where $x$ is the quantity of the examined good, $r$ is the expenditure for it and $m$ is buyer's purchasing capacity.. ${ }^{1}$ Moreover, we assume that functions $u($.$) and g($.$) are differentiable, non decreasing and$ concave, so the indifference curves are non-increasing and convex.

The indifference curve of the $i$-th buyer, on which the monopolist can choose any point that he likes (and he can do so for all $n$ buyers), is represented by $u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)=g_{i}\left(m_{i}\right)$, where $x_{i}$ is the quantity of the good sold to the $i$-th buyer, $m_{i}$ is his purchasing capacity (given in monetary terms) and $r_{i}$ is the amount paid by the buyer to obtain quantity $x_{i}$ of the good. For simplicity we assume that $u_{i}(0)=0$. The monopolist can choose a pair $\left(x_{i}, r_{i}\right)$ under the constraint $u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)=g_{i}\left(m_{i}\right)$.

The revenue of the monopolist is equal to $\sum_{i=1}^{n} r_{i}$. The monopolist maximizes his revenue for each quantity $X=\sum_{i=1}^{n} x_{i}$. Therefore, the choice of the monopolist is a solution to the problem $\max _{\left(x_{i}, r_{i}\right)_{i=1}^{n}} \sum_{i=1}^{n} r_{i}$ subject to the constraints $X=\sum_{i=1}^{n} x_{i}$ and $u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)=g_{i}\left(m_{i}\right)$ for $i=1, \ldots, n$. The Lagrangian function is

$$
\begin{aligned}
& \mathrm{L}\left(x_{1}, \ldots, x_{n}, r_{1}, \ldots,\right. \\
& \left.r_{n}, \lambda, \mu_{1}, \ldots, \mu_{n}\right)= \\
& \quad \sum_{i=1}^{n} r_{i}+\lambda\left(X-\sum_{i=1}^{n} x_{i}\right)+\sum_{i=1}^{n} \mu_{i}\left(u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)-g_{i}\left(m_{i}\right)\right)
\end{aligned}
$$

From the first-order conditions for internal solution

$$
\begin{array}{lll}
\mu_{i} \mathrm{D}_{x_{i}} u_{i}\left(x_{i}\right)=\lambda, & \mu_{i} \mathrm{D}_{m_{i}} g_{i}\left(m_{i}-r_{i}\right)=1, & i=1, \ldots, n \\
X=\sum_{i=1}^{n} x_{i}, & u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)=g_{i}\left(m_{i}\right), & i=1, \ldots, n
\end{array}
$$

we get

$$
\frac{\mathrm{D}_{x_{i}} u_{i}\left(x_{i}\right)}{\mathrm{D}_{m_{i}} g_{i}\left(m_{i}-r_{i}\right)}=\lambda, \quad u_{i}\left(x_{i}\right)+g_{i}\left(m_{i}-r_{i}\right)=g_{i}\left(m_{i}\right), \quad i=1, \ldots, n
$$

Therefore, these conditions require that the points chosen by the monopolist on the indifference curves have the same marginal rate of substitution for all buyers. As a result we get the revenue function $R(X)$ $=\max _{\left(x_{i}, r_{i}\right)_{i=1}^{n}} \sum_{i=1}^{n} r_{i}$, where $\mathrm{D}_{X} R(X)=\lambda$ (since, by differentiating the preceding conditions, we get $\mathrm{d} R=\sum_{i=1}^{n} \mathrm{~d} r_{i}, \mathrm{~d} X=\sum_{i=1}^{n} \mathrm{~d} x_{i}$ and $\mathrm{d} r_{i}=\frac{\mathrm{D}_{x_{i}} u_{i}\left(x_{i}\right)}{\mathrm{D}_{m_{i}} g_{i}\left(m_{i}-r_{i}\right)} \mathrm{d} x_{i}$
for $i=1, \ldots, n$ ) and $\mathrm{D}_{X}^{2} R(X) \leq 0$ (because we assumed that buyer's indifference curves are convex). The quantity to be sold $X^{*}$ is chosen by the monopolist by solving profit maximization problem, in other words by

[^0]equalizing marginal revenue with marginal cost. Doing so we get the following equilibrium conditions
$$
M C\left(X^{*}\right)=M R\left(X^{*}\right)=\frac{\mathrm{D}_{x_{i}} u_{i}\left(x_{i}^{*}\right)}{\mathrm{D}_{m_{i}} g_{i}\left(m_{i}-r_{i}^{*}\right)}=\lambda^{*}
$$
with
$X^{*}=\sum_{i=1}^{n} x_{i}^{*}, R\left(X^{*}\right)=\sum_{i=1}^{n} r_{i}^{*}, u_{i}\left(x_{i}{ }^{*}\right)+g_{i}\left(m_{i}-r_{i}{ }^{*}\right)=g_{i}\left(m_{i}\right)$, for $i=1, \ldots, n$, which imply equality between the marginals for all agents (marginal cost and revenue of the monopolist, marginal rates of substitution for the buyers). The common value $\lambda^{*}$ indicates the marginal price of the examined good.


Figure 10.23

In Figure 10.23 we depict equilibrium of the monopoly with firstdegree price discrimination when all the consumers are identical. Left part of the figure represents in scale 1:n the production set that coincides in this case with the cost function of the producer (quantity produced is on the vertical axis and cost of the production is on the horizontal axis). Right part of the figure represents Edgeworth-Pareto diagram, with the indifference curves of the producer (isoprofit curves) generated by transposition of the cost function. The producer chooses the production level on the initial indifference curve of the consumers such that it maximizes his profit. The tangency point depicted in the figure determines the producer's choice.

Note that the marginal price $\lambda^{*}$, the quantity produced $X^{*}$ and the quantity purchased $x_{i}{ }^{*}$ by the buyers in the equilibrium with monopoly with first-degree price discrimination coincide with the equilibrium values in the competitive equilibrium (consumer spending and, as a result, the revenue of the producer differ in these two equilibria however). Indeed, with unique price and agents acting as price-takers, the competitive equilibrium is
determined by conditions $X^{*}=\sum_{i=1}^{n} x_{i}{ }^{*}, M C\left(X^{*}\right)=\frac{\mathrm{D}_{x_{i}} u_{i}\left(x_{i}{ }^{*}\right)}{\mathrm{D}_{m_{i}} g_{i}\left(m_{i}-p^{*} x_{i}{ }^{*}\right)}=p^{*}$ for $i=1, \ldots, n$.
b) Monopoly with second-degree price discrimination. In this case monopolist knows the typology of the buyers but he is not able to recognize the type of each individual buyer. For example, he knows that there are two types of buyer and he knows their preferences but he does not know which type of the consumer is John. (You can imagine that the monopolist is the manager of the only theatre in a city with two types of audience - passionate and occasional. The manager is aware of the existence of the two types but cannot recognize the type of the client when he shows up at the ticket office). The monopolist can in some cases exploit the market by proposing different contracts, such that each of them is preferred by one type of the consumer. In this way he can sort the buyers. Each offered contract is a pair of the quantity to be bought and its corresponding cost: that is, $h$-th contract is a pair $\left(x_{h}, r_{h}\right){ }^{2}$

Here we consider a case with only two types of the buyer. We assume, just like before, that before the purchase each buyer has no quantity of the good examined and that the purchase of other goods does not depend on conditions in the market for the good in question with exception for the expenditure for the good in question. The two types of the buyer have, respectively, the utility functions $u_{1}\left(x_{1}\right)+g_{1}\left(m_{1}-r_{1}\right)$ and $u_{2}\left(x_{2}\right)+g_{2}\left(m_{2}-r_{2}\right)$, where $u_{1}(0)=u_{2}(0)=0$ and the functions $u_{1}(),. g_{1}(),. u_{2}($.$) and g_{2}($.$) are non$ decreasing, concave and differentiable. Moreover, we assume that one of types of the consumer, let's say the first type, is more likely to buy the good than the other type. That is, whenever the second type likes a contract ( $x, r$ ), the first type likes it as well, i.e. $u_{2}(x)+g_{2}\left(m_{2}-r\right) \geq g_{2}\left(m_{2}\right)$ implies $u_{1}(x)+g_{1}\left(m_{1}-r\right) \geq g_{1}\left(m_{1}\right)$. In what follows, we consider the possibility that for every possible contract the first type of the consumer has a higher marginal rate of substitution than the second type of the consumer, that is $\frac{\mathrm{D}_{\chi} u_{1}(x)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r\right)}>\frac{\mathrm{D}_{\chi} u_{2}(x)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r\right)}$, so he is willing to pay more for the good than the second type of the consumer.

Under these assumptions, the monopolist knows that every contract that results in purchase by the second type of the buyer will also result in purchase by the first type of the buyer. However, there can be contracts that attract buyers of the first type but not the ones of the second type. Therefore, the monopolist can introduce screening of the buyers. To the second type of buyer he can propose a contract that if accepted makes him just a little bit better off than he is in the initial situation, that is, neglecting this little bit, a contract $\left(x_{2}, r_{2}\right)$ such that $u_{2}\left(x_{2}\right)+g_{2}\left(m_{2}-r_{2}\right)=g_{2}\left(m_{2}\right)$. To the first type of

[^1]buyer he can propose a contract a little bit better from his perspective, that is a contract a little bit better than $\left(x_{1}, r_{1}\right)$ which makes consumer of the first type indifferent to the contract proposed to the second type of consumers, that is a contract such that $\left.u_{1}\left(x_{1}\right)+g_{1}\left(m_{1}-r_{1}\right)=u_{1}\left(x_{2}\right)+g_{1}\left(m_{1}-r_{2}\right)\right)$. Screening occurs if the contract $\left(x_{1}, r_{1}\right)$ is not preferred by the consumers of the second type (that is $u_{2}\left(x_{1}\right)+g_{2}\left(m_{2}-r_{1}\right)<g_{2}\left(m_{2}\right)$ ). In such a case, type one consumers choose a contract (just a little bit better than) ( $x_{1}, r_{1}$ ) and type two consumers choose a contract (just a little bit better than) ( $x_{2}, r_{2}$ ). The monopolist chooses the contracts to maximize his profit subject to the described constraints. Let $N_{1}$ and $N_{2}$ be the number of consumers of the corresponding type and let $C(Q)$ be the cost function, where $Q=N_{1} x_{1}+N_{2} x_{2}$. Then the problem that we want to solve becomes
$$
\max _{x_{1}, x_{2}, r_{1}, r_{2}} N_{1} r_{1}+N_{2} r_{2}-C\left(N_{1} x_{1}+N_{2} x_{2}\right)
$$
subject to
$u_{1}\left(x_{1}\right)+g_{1}\left(m_{1}-r_{1}\right)=u_{1}\left(x_{2}\right)+g_{1}\left(m_{1}-r_{2}\right)$ and $u_{2}\left(x_{2}\right)+g_{2}\left(m_{2}-r_{2}\right)=g_{2}\left(m_{2}\right)$, provided that the solution satisfies the condition $u_{2}\left(x_{1}\right)+g_{2}\left(m_{2}-r_{1}\right)<g_{2}\left(m_{2}\right)$. The Lagrangian is
$\mathrm{L}\left(x_{1}, x_{2}, r_{1}, r_{2}, \lambda_{1}, \lambda_{2}\right)=\quad N_{1} r_{1}+N_{2} r_{2}-C\left(N_{1} x_{1}+N_{2} x_{2}\right)+$
$\lambda_{1}\left(u_{1}\left(x_{1}\right)+g_{1}\left(m_{1}-r_{1}\right)-u_{1}\left(x_{2}\right)-g_{1}\left(m_{1}-r_{2}\right)\right)+\lambda_{2}\left(u_{2}\left(x_{2}\right)+g_{2}\left(m_{2}-r_{2}\right)-g_{2}\left(m_{2}\right)\right)$,
which gives rise to the following first order conditions, where $Q^{*}=$ $N_{1 X_{1}}{ }^{*}+N_{2} X_{2}{ }^{*}$ :
$N_{1} M C\left(Q^{*}\right)=\lambda_{1} \mathrm{D}_{x} u_{1}\left(x_{1}{ }^{*}\right), \quad N_{2} M C\left(Q^{*}\right)+\lambda_{1} \mathrm{D}_{x} u_{1}\left(x_{2}{ }^{*}\right)=\lambda_{2} \mathrm{D}_{x} u_{2}\left(x_{2}{ }^{*}\right)$,
$N_{1}=\lambda_{1} \mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}^{*}\right), \quad N_{2}+\lambda_{1} \mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}{ }^{*}\right)=\lambda_{2} \mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)$,
$u_{1}\left(x_{1}{ }^{*}\right)+g_{1}\left(m_{1}-r_{1}{ }^{*}\right)=u_{1}\left(x_{2}{ }^{*}\right)+g_{1}\left(m_{1}-r_{2}{ }^{*}\right), \quad u_{2}\left(x_{2}{ }^{*}\right)+g_{2}\left(m_{2}-r_{2}{ }^{*}\right)=g_{2}\left(m_{2}\right)$.
These conditions imply
\[

$$
\begin{aligned}
& \frac{\mathrm{D}_{\chi} u_{1}\left(x_{1}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}^{*}\right)}=M C\left(Q^{*}\right), \\
& \frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}^{*}\right)}=M C\left(Q^{*}\right) \frac{N_{2}+N_{1} \frac{\mathrm{D}_{\chi} u_{1}\left(x_{2}^{*}\right)}{\mathrm{D}_{\chi} u_{1}\left(x_{1}{ }^{*}\right)}}{N_{2}+N_{1} \frac{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}^{*}\right)}}
\end{aligned}
$$
\]

The consumers of the first type (the ones more likely to buy) receive a favorable contract (that substantially increases their utility) and their marginal rate of substitution is equal to the marginal cost. The second type of buyers receives a less favorable contract (that does not substantially increase their utility) and their marginal rate of substitution may differ from the marginal cost. In particular, since the preceding relationships imply the following equality

$$
N_{2}\left(\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}-\frac{\mathrm{D}_{\chi} u_{1}\left(x_{1}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}^{*}\right)}\right)=
$$

$$
=N_{1} \frac{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}{ }^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}{ }^{*}\right)}\left(\frac{\mathrm{D}_{\chi} u_{1}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}^{*}\right)}-\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}\right)
$$

the following inequalities can hold

$$
\frac{\mathrm{D}_{\chi} u_{1}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}{ }^{*}\right)}>\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}{ }^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}>\frac{\mathrm{D}_{\chi} u_{1}\left(x_{1}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}{ }^{*}\right)}
$$

or the opposite ones

$$
\frac{\mathrm{D}_{\chi} u_{1}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{2}{ }^{*}\right)}<\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}<\frac{\mathrm{D}_{\chi} u_{1}\left(x_{1}^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}^{*}\right)}
$$

In the first case we get $\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}{ }^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}>M C\left(Q^{*}\right)$. Moreover, $x_{1}{ }^{*}>x_{2}{ }^{*}$ since $\left(x_{1}{ }^{*}, m_{1}-r_{1}{ }^{*}\right)$ and $\left(x_{2}{ }^{*}, m_{1}-r_{2}{ }^{*}\right)$ are on the same convex indifference curve $u_{1}\left(x_{1}{ }^{*}\right)+g_{1}\left(m_{1}-r_{1}{ }^{*}\right)=u_{1}\left(x_{2}{ }^{*}\right)+g_{1}\left(m_{1}-r_{2}{ }^{*}\right)$. In the second case, all the inequalities are reversed, that is $\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}{ }^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}<M C\left(Q^{*}\right)$ and $x_{1}{ }^{*}<x_{2}{ }^{*}$.

If the condition $\frac{\mathrm{D}_{\chi} u_{1}(x)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r\right)}>\frac{\mathrm{D}_{\chi} u_{2}(x)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r\right)}$ (that says that the consumers of the first type are willing to pay more for the good) holds, then, on one hand, only the first case can occur and, on the other hand, the condition $u_{2}\left(x_{1}{ }^{*}\right)+g_{2}\left(m_{2}-r_{1}{ }^{*}\right)<g_{2}\left(m_{2}\right)$ is satisfied. ${ }^{3}$ Therefore, the contract ( $x_{1}, r_{1}$ ) does not attract consumers of the second type and, as a result, it is profitable for the monopolist to offer it and screening occurs. In this case, the contract $\left(x_{1}{ }^{*}, r_{1}{ }^{*}\right)$ is equivalent to a ticket at price $t_{1}{ }^{*}$ that gives right to purchase the good (without the option to resale it) at price $p_{1}{ }^{*}$ and the contract $\left(x_{2}{ }^{*}, r_{2}{ }^{*}\right)$ is equivalent to the ticket at price $t_{2} *$ that gives right to purchase the good at price $p_{2}{ }^{*}$. Equivalence for the first contract comes from conditions $p_{1}{ }^{*}=\frac{\mathrm{D}_{\chi} u_{1}\left(x_{1}{ }^{*}\right)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r_{1}{ }^{*}\right)}, t_{1} *=r_{1}^{*}-p_{1}{ }^{*} x_{1} *$ and, for the second contract, from conditions $p_{2}{ }^{*}=\frac{\mathrm{D}_{\chi} u_{2}\left(x_{2}{ }^{*}\right)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r_{2}{ }^{*}\right)}$ and $t_{2}{ }^{*}=r_{2}{ }^{*}-p_{2}{ }^{*} x_{2} *$. The convexity of the indifference curves and the conditions examined for this case, imply $t_{1}{ }^{*}>t_{2}{ }^{*}>0$ and $M C\left(Q^{*}\right)=p_{1}{ }^{*}<p_{2}{ }^{*}$. Therefore, buyers of the second type prefer to buy less expensive ticket that gives them the right to buy the good at a higher price and the buyers of the first type prefer to do

$$
{ }^{3} \text { Since } \frac{\mathrm{D}_{\chi} u_{1}(x)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r\right)}>\frac{\mathrm{D}_{\chi} u_{2}(x)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r\right)} \text {, then, for every } x \in\left[x_{2}^{*}, x_{1}{ }^{*}\right] \text {, in every point }
$$ $(x, r)$ belonging to the indifference curve $u_{1}(x)+g_{1}\left(m_{1}-r\right)=u_{1}\left(x_{2}{ }^{*}\right)+g_{1}\left(m_{1}-r_{2}^{*}\right)$, the slope of this indifference curve is (in absolute values) higher than the slope of the indifference curves $U_{2}=u_{2}(x)+g_{2}\left(m_{2}-r\right)$. Therefore, when $x$ increases, the utility $u_{2}$ decreases. As a result we get $g_{2}\left(m_{2}\right)=u_{2}\left(x_{2}{ }^{*}\right)+g_{2}\left(m_{2}-r_{2}{ }^{*}\right)>u_{2}\left(x_{1}{ }^{*}\right)+g_{2}\left(m_{2}-r_{1}{ }^{*}\right)$.

the opposite. This type of price (composed of a fixed cost and a cost proportional to the quantity purchased) is an example of non linear prices. ${ }^{4}$


Figure 10.24

Figure 10.24 depicts the described situation (in this figure indifference curves of two types of the buyer are translated along the vertical axis such that the point $\left(0, m_{1}\right)$ of the first type coincides with the point $\left(0, m_{2}\right)$ of the second type).

If, on the contrary, $\frac{\mathrm{D}_{\chi} u_{1}(x)}{\mathrm{D}_{m} g_{1}\left(m_{1}-r\right)}<\frac{\mathrm{D}_{\chi} u_{2}(x)}{\mathrm{D}_{m} g_{2}\left(m_{2}-r\right)}$, then screening may be unprofitable for the monopolist. In such a case, he prefers to offer the same sale conditions to both types of the buyer.
c) Monopoly with third-degree price discrimination. In this case the monopolist can distinguish between groups of buyers and he knows demand curve of every group. Moreover, buyers cannot resale the good to buyers outside their group. Therefore, the monopolist sells the good on separate markets and for each of them he chooses the monopoly price. (For example, the monopolist sells its product in different countries at a different price for each country, or sells the good at a price that depends on the age of the buyer, etc.).

Let there be two distinguishable groups of buyers, characterized, respectively, by the demand functions $Q_{1}=D_{1}\left(p_{1}\right)$ and $Q_{2}=D_{2}\left(p_{2}\right)$, and let the monopolist have the cost function $C\left(Q_{1}+Q_{2}\right)$. Let the revenue functions in the two markets be denoted with $R_{1}\left(Q_{1}\right)$ and $R_{2}\left(Q_{2}\right)$ (where $R_{1}\left(Q_{1}\right)=$

[^2]$Q_{1} D_{1}^{-1}\left(Q_{1}\right)$ and $\left.R_{2}\left(Q_{2}\right)=Q_{2} D_{2}^{-1}\left(Q_{2}\right)\right)$. Profit maximization gives rise to the following first order conditions (that equalize marginal cost to marginal revenue in each market)
$M C\left(Q_{1}{ }^{*}+Q_{2}{ }^{*}\right)=M R_{1}\left(Q_{1}{ }^{*}\right)=M R_{2}\left(Q_{2}{ }^{*}\right), \quad Q_{1}{ }^{*}=D_{1}\left(p_{1}{ }^{*}\right), \quad Q_{2}{ }^{*}=D_{2}\left(p_{2}{ }^{*}\right)$
and the second order conditions
$$
\mathrm{D}_{Q} M C\left(Q_{1}{ }^{*}+Q_{2}^{*}\right) \geq \mathrm{D}_{Q} M R_{1}\left(Q_{1}{ }^{*}\right), \quad \mathrm{D}_{Q} M C\left(Q_{1}{ }^{*}+Q_{2}^{*}\right) \geq \mathrm{D}_{Q} M R_{2}\left(Q_{2}^{*}\right) .
$$

Figure 10.25 depicts equilibrium with the third-degree price discrimination. The curve that determines the quantity produced $Q_{1}{ }^{*}+Q_{2}{ }^{*}$ (at the intersection point with the marginal cost curve) is obtained by summation of the marginal revenue curves $M R_{1}\left(Q_{1}\right)$ and $M R_{2}\left(Q_{2}\right)$.


Figure 10.25

If we describe marginal revenue in terms of demand elasticity we get $M R=p\left(1-\frac{1}{\varepsilon}\right)$. Therefore, the first order condition $M R_{1}\left(Q_{1}{ }^{*}\right)=M R_{2}\left(Q_{2}{ }^{*}\right)$ requires $p_{1}\left(1-\frac{1}{\varepsilon_{1}}\right)=p_{2}\left(1-\frac{1}{\varepsilon_{2}}\right)$. This relationship implies that monopolist chooses higher price in the markets where demand elasticity is lower.

### 10.11 Monopolistic competition equilibrium

This type of market (introduced by Chamberlin, 1933) has features intermediate between perfect competition and monopoly. Like in monopoly, sellers are price-makers, because each of them sells a different good. Like in perfect competition there are many sellers and free entry. What is particular
about this type of market is the presence of differentiated products that are heterogeneous but easily substituted. The examples of such markets are market for restaurants, retail markets, wine market, etc. There is a parametric (instead of strategic) behavior if choice of each single seller does not influence the profit of other sellers. Nevertheless, the substitutability of the goods means that the demand for each good is a function of the average price of all similar goods (for which choice of a single seller is negligible).

The demand for the good produced by $j$-th firm can be represented by a function $q_{j}=d_{j}\left(p_{j}, P\right)$, where $P$ denotes average price of similar goods. Since these goods are substitutes we get $\frac{\partial d_{j}\left(p_{j}, P\right)}{\partial P}>0$. In a short period (and with a given number of firms), each seller takes $P$ as given (because other sellers do not react to his choice) and determine the price as in monopoly with unique pricing. Therefore, we get the following conditions
$\mathrm{D}_{q} R_{j}\left(q_{j}{ }^{*}, P\right)=\mathrm{D}_{q} C_{j}\left(q_{j}{ }^{*}\right), \quad \mathrm{D}_{q q}^{2} R_{j}\left(q_{j}{ }^{*}, P\right) \leq \mathrm{D}_{q}^{2} C_{j}\left(q_{j}{ }^{*}\right), \quad q_{j}{ }^{*}=d_{j}\left(p_{j}{ }^{*}, P\right)$,
and Figure 10.19 representation holds.
Free entry drives the very long term equilibrium in which every firm makes zero profit. For example, if there was a positive profit, the entry of new firms would increase quantity offered and reduces average price. Therefore, demand curve for each company would move down reducing the profit until it is equal to zero and new firms stop entering. The situation of the $j$-th firm is represented in Figure 10.26.


Figure 10.26

Note that since the demand curve is decreasing, zero profit condition implies that the average cost is decreasing at the quantity produced $q_{j}{ }^{*}$. This
means that the firm can produce more of the good at a lower average cost. Therefore, monopolistic competition leads to excess of production capacity, while in the very long term free competition each firm produces at the minimal average cost and in the monopoly with unique price firm produces at an average cost that can be larger or equal to the minimal cost. Nevertheless, while in these types of market the good is homogenous, in the monopolistic competition the goods can be differentiated. This is advantageous for the consumers, who can choose from the variety of goods in the market the one they like most.


[^0]:    ${ }^{1}$ Indeed, let the utility function be of the following type $u\left(x_{1}\right)+v\left(x_{2}, \ldots, x_{k}\right)$, let the good one be the good in question, and let the agent be a price-taker for the other goods. For these goods we have demand function $x_{h}=d_{h}\left(p_{2}, \ldots, p_{k}, m-r_{1}\right)$, where $m$ is the purchasing capacity, for every pair ( $x_{1}, r_{1}$ ). Therefore, the utility function becomes $u\left(x_{1}\right)+g\left(p_{2}, \ldots, p_{k}, m-r_{1}\right)$, where $g($.$) is the indirect utility relative to the other goods.$ Taking into account given prices of the other goods, we get the above utility function.

[^1]:    ${ }^{2}$ In this paragraph we examine a case in which monopolist produces a homogenous good. If he produces differentiated products, as in the case analyzed in the next paragraph, then the contract can include more goods and different contracts can include different goods.

[^2]:    ${ }^{4}$ Non linear prices are common for natural monopolies (like for example the supply of electricity). The reasons are different than the ones presented here and will become clear in Paragraph 10.12, when we examine efficiency.

