INTRODUCTION TO REAL ANALYSIS II

PhD in Statistics, Bocconi University

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Course Description

The course provides an introduction to the theory of Optimal Transport, in the context of Convex and Functional Analysis. Optimal Transport deals with the problem to find the best strategy to move a given distribution of masses (or goods, or physical quantities…) to an assigned configuration, in order to minimize a cost depending on the initial and final position. Starting from the pioneering and famous work by Monge (1781) and after the fundamental contribution by Kantorovich in the first half of the last century, this fascinating subject have been investigated under various point of views, and has grown up to form a rich and beautiful theory with many applications to statistics, probability, convex and functional analysis, calculus of variations, PDE's and geometry. The aim of the course is to provide the principal results of Optimal Transport, within the broader framework of Measure Theory, Convex and Functional analysis.

Syllabus

- The discrete case and finite dimensional linear programming.
- Basic results of convex duality (Von Neumann minimax theorem)
- A brief review of measure theoretic tools (weak convergence, Prokhorov theorem, disintegration).
- Modern formulation of Optimal Transport. Kantorovich duality and optimality conditions in general Euclidean and Topological Spaces.
- Optimal Transport in the real line, monotone rearrangement of a measure.
- The case "cost=distance".
- The case of the quadratic cost in finite dimensional Euclidean spaces, Brenier map.
- The metric viewpoint: weak convergence and distances between probability measures.
- The dynamic approach of Benamou-Brenier: continuity equation and curves of probability measures.
- Convex functionals of measures, Entropic relaxation, unbalanced Optimal Transport and Hellinger-Kantorovich distance.

Textbooks

Filippo Santambrogio, *Optimal Transport for Applied Mathematicians*, Birkhauser (2015)
Cédric Villani, *Optimal Transport. Old and New*, Springer (2009)
Luigi Ambrosio, Nicola Gigli, Giuseppe Savaré, *Gradient Flows. In Metric Spaces and in the Space of Probability Measures*, Birkhauser (2008)
Gabriel Peyré, Macro Cuturi: *Computational Optimal Transport*, https://optimaltransport.github.io/book/

Exam

Written exam