Towards unified Bayesian estimation of diffusions

Frank van der Meulen[1], Moritz Schauer[2], Omiros Papaspiliopoulos[3], Harry van Zanten[4]

[1] *Delft University of Technology, Netherlands*
[2] *University of Leiden, Netherlands*
[3] *Universitat Pompeu Fabra, Spain*
[4] *University of Amsterdam, Netherlands*

Abstract

Bayesian estimation of discretely observed diffusions has received much attention over the past two decades. If the data generating SDE is given by

$$dX_t = b(t; X_t)dt + \sigma(t; X_t)dW_t;$$

then it is commonly assumed that the matrix $a = \sigma\sigma'$ is non-degenerate. In applications, this uniform ellipticity assumption is not always met. As an example, consider the two dimensional system where the first component is the integral of the second:

$$dX_t = \begin{bmatrix} X_{t2} \\ g(t; X_t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ \gamma(t; X_t) \end{bmatrix} dWt.$$

This is an example of an integrated diffusion, or more generally, of a hypo-elliptic diffusion. To obtain samples from the posterior, it is common to use data-augmentation, where latent diffusion bridges in between discrete time observations are simulated. Unfortunately, many methods for simulating bridges crucially depend on the uniform ellipticity assumption. While hypo-elliptic diffusions are interesting from an application perspective, no general solution for Bayesian estimation has appeared in the literature. We explain how the approach of Schauer et al. can be adapted and generalised for this purpose. Although various non-trivial mathematical problems pop-up compared to the uniformly elliptic case, it turns out that with some modifications, the approach of Schauer et al. can be used for hypo-elliptic diffusions as well. This results in a unified method for Bayesian estimation in the sense that it is applicable to both the uniformly- and hypo-elliptic diffusions.

Schauer, M. and Van der Meulen, F. H. and Van Zanten, J. H. (2016) *Guided proposals for simulating multi-dimensional diffusion bridges.* Accepted for publication in Bernoulli, ArXiv e-prints 1311.3606.